Universal Structures in Mathematics and Computing 2016

28 – 29 June 2016
La Trobe University

Welcome to Universal Structures in Mathematics and Computing. The first Universal Structures in Mathematics and Computing was held at the Australian National University in 2007. It stated

> "This workshop aims to bring together researchers working in category theory, universal algebra, logic and their applications to computer science in order to highlight recent advances in these fields and to facilitate dialogue between the different camps. Of particular interest is work which spans two or more of these areas."

USMaC 2016 is an attempt to repeat this, and more generally bring together (relatively) local researchers working in mathematical foundations of computer science.

logo by Lucy Ham

information

talks

All talks will be held in Teaching Room 3 of the City Campus of La Trobe University: level 20 of 360 Collins Street, Melbourne CBD.

internet access

There is wireless internet throughout the campus, which you should be able to use if you have access to *eduroam*.

coffee/tea

During the morning and afternoon tea breaks, the workshop will provide sweet and savoury refreshments, as well as juice, tea and coffee. These will be served in the "Student Lounge" area to the left of Teaching Room 3 (as you leave it). There is a coffee machine with push-button operation (and what appears to be freshish beans, but this might be an optical illusion). Please note that the Student Lounge is a shared area.

lunch

The majority of Melbourne CBD is within viable lunch-time walking distance. Some possible nearby areas to try out are as follows.

- **Degraves Street.** Head east across Elizabeth Street, then turn south down Centre Way (a little arcade/opening not far after Elizabeth Street is crossed). Alternatively, head left into Block Arcade until it becomes Block Place just before Bourke Street. Between Queen and Elizabeth Streets.
- **Hardware Lane precinct.** Directly north of the workshop venue: just after Bourke Street is crossed.
- **Little Collins Street.** Even just around the back of 360 Collins Street you will find plenty of food (head a few metres east beyond the building, then take the first little laneway "Equitable Place").

coffee snobs

Nearby best speciality coffee shops (rankings from Beanhunter for greater Melbourne):

(#5) LB2, Speciality Coffee, 2 Gallagher Place. East of Kings St, on Ltl Collins.
(#6) Patricia, 493–495 Ltl Bourke Street. Just east of William St.
(#10) Cup of Truth, 12 Campbell Arcade, Degraves Street Subway. Hole-in-Wall.
(#13) Little Bean Blue, 15 Ltl Collins St. Just west of Spring St.
(#23) Brother Baba Budan, 359 Ltl Bourke St. Just west of Elizabeth St.

sponsors

Thanks to La Trobe University and the Discipline Research Program in Mathematics and Computing, which have provided funding for this event.

questions?

Feel free to ask us, or any of the other locals, if you need assistance during the conference.

Marcel Jackson and Tomasz Kowalski

Department of Mathematics and Statistics, La Trobe University

conference dinner

The workshop dinner is on Tuesday, 6.00pm, at *The Crafty Squire*, 127 Russell Street, in the CBD. We've booked the Brewers Lounge, so don't get lost in the general bar area! The Brewers Lounge should be next to the giant brewing apparatus. See the website the-craftysquire.com.au for details.

It's about 11 minutes walk (800m) from the Collins Street campus: walk around 600m east (towards Elizabeth Street, then Swanston Street) and then left (north) along Russell Street for about 150m. It's on the left (west) side of the street.

The event will be fairly informal: we're ordering off the general menu, and no-one will be forced to order at 6PM sharp. Perhaps some of the local brews might be a better way to start?

abstracts

1. The simplest (consistent?) foundation for logic and mathematics?

Martin Bunder (University of Wollongong)
Prof Martin Bunder

14:30 Wed 29 June 2016

The system has just 3 constants, no variables and not many axioms and rules. I will show briefly how logic of arbitrarily high order and a lot of mathematicss can be developed in this system. Large subsystems have been proved consistent but these are not enough to develop all the mathematical applications. I have worked recently on this without complete success, but I'll mention some of the things I've tried and some partial results.

2. An abstract definition of a restricted Priestley duality with applications to discriminator varieties

Brian Davey (La Trobe University)
Prof Brian Davey and Asha Gair

16:30 Tue 28 June 2016

Anyone who has ever worked with a variety $\mathcal{A}$ of algebras with a reduct in the variety of bounded distributive lattices will know a restricted Priestley duality when they meet one—but until now there has been no abstract definition. Here we provide one. After deriving some basic properties of a restricted Priestley dual category $\mathcal{X}$ of such a variety, we give a characterisation, in terms of $\mathcal{X}$, of finitely generated discriminator subvarieties of $\mathcal{A}$.

As an application of the theory we give a characterisation of discriminator varieties of Cornish algebras.

3. Pattern languages and solutions to equations in groups

Murray Elder (The University of Newcastle)
Dr Murray Elder

14:30 Tue 28 June 2016

I will explain what pattern languages are and how my work on solutions to equations in free groups (with Ciobanu and Diekert) answers an open problem about them.

4. Spatial Logic of Tangled Closure and Derivative Operators

Rob Goldblatt (Victoria University of Wellington)
Prof Rob Goldblatt

12:00 Wed 29 June 2016

The tangled closure of a collection of sets is the largest set in which each member of the collection is dense. This operation models a generalised modality that was introduced by Dawar and Otto to characterise the bisimulation-invariant fragments of both first-order logic and monadic second-order logic, and the modal mu-calculus, over over certain classes of finite transitive structures.

This talk surveys joint work with Ian Hodkinson on interpretations of the tangle modality, including a variant in which topological closure is replaced by the derivative (= set of limit points) operation. We prove the finite model property for, and provide complete axiomatisations of, the logics of a range of topological spaces in a number of languages, some with the universal modality. This includes results for all Euclidean spaces, and all zero-dimensional dense-in-themselves metric spaces. The methods used involve new kinds of 'dissections' of metric spaces in the sense of McKinsey and Tarski.

5. Relation-Algebraic Verification of Prim's Minimum Spanning Tree Algorithm

Walter Guttmann (University of Canterbury)
Dr Walter Guttmann

12:00 Tue 28 June 2016

Relation-algebraic methods have been used to develop algorithms for unweighted graphs. This works well because unweighted graphs can be directly represented as relations. We generalise relation algebras and Kleene algebras to model weighted graphs, which do not have direct representations as binary relations. Using the generalised algebras and a few extensions, we prove the correctness of Prim's minimum spanning tree algorithm. The proof is formally verified in Isabelle/HOL, including the overall Hoare-logic argument, algebraic theories, calculations and models.

6. Universal Algebra of Constraint Satisfaction Problems

Marcel Jackson (La Trobe University)
Dr Marcel Jackson

15:00 Wed 29 June 2016

This talk will provide a short survey of the universal algebra of constraint satisfaction problems over fixed templates.

7. Universal updates arising among bidirectional transformations

Michael Johnson (Macquarie University)
Prof Michael Johnson

09:30 Tue 28 June 2016

Bidirectional transformations are a computer science notion of rising importance because of the growing need to synchronise disparate data sources. As mathematical structures bidirectional transformations, when presented as so-called set-based “lenses”, are algebras for a monad on a slice category. A more detailed analysis of the inputs required for effective system interoperation leads to a refined notion of lens which turns out to be an algebra for a correspondingly refined monad. Pleasingly, and for many, surprisingly, the action of that algebra produces universal updates – updates which satisfy a very desirable (in applications) universal property sometimes called “least change”.

8. Network satisfaction problem over McKenzie's algebra

Tomasz Kowalski (La Trobe University)
Dr Tomasz Kowalski

17:00 Tue 28 June 2016

A (general) network satisfaction problem (GEN-SAT) over a relation algebra A is the following: given an (abstract) relation algebra A and a finite graph N labelled by the elements of A, is there a representation of A into which N maps homomorphically. GEN-SAT also applies to certain representations weaker than the usual (strong) one, in particular, to qualitative representations.

About 40 years ago, McKenzie found a small relation algebra which is not representable in the strong sense (in fact his algebra is of minimal size with this property). However, it is qualitatively representable (over posets of width 2). I will show that GEN-SAT over McKenzie algebra is NP-complete. Essentially the same argument applies to many other qualitatively representable algebras.

9. Theory of Relational Calculus and its formalization

Yoshihiro Mizoguchi (Kyushu University)
Assoc Yoshihiro Mizoguchi

11:00 Wed 29 June 2016

There are many network structures (relations between certain objects) considered in applications of mathematics for industry. We use many calculations of numbers and equations of numbers in mathematical analysis. But we seldom use calculations of network structures or equations of relational structures. On the other hand, a sufficiently developed theory of relations has been existing for a long while. In this talk, we review those theory of relations from the view point of a computation. We show an elementary theory of relations and its formalization in Coq, a proof assistant system. Further, we introduce automatic proving procedures (tactics) for our formalization of the theory of relational calculus.

10. Relational Approaches to Physical Reasoning

Jochen Renz (Australian National University)
Prof Jochen Renz

09:30 Wed 29 June 2016

We study problems faced by AI agents when interacting with the physical world. This includes analysing an observed situation and inferring properties or suitable physical actions. Our approaches are mostly based on hybrid reasoning and combine spatial relation algebras with quantitative spatial information obtained from cameras and other sensors. We present a number of difficult physical reasoning problems and their solutions which showcase the usefulness of relational reasoning in practical applications.

11. Constellations: Arrows Without Targets.

Tim Stokes (University of Waikato)
Dr Tim Stokes

11:00 Tue 28 June 2016

Constellations are partial algebras that are one-sided generalisations of categories. Categories model classes of objects together with suitably defined mappings between them. Each mapping, or arrow, has a domain and codomain (source and target), and composition of mappings $f \cdot g$ is defined precisely when the codomain of f coincides with the domain of g. An alternative notion of composition arises if one only requires the codomain of f to be a *subset* of the domain of g. When this is done, precise information about codomains is no longer needed, and "arrows" have sources but no targets. This is more natural in many examples, for example all mappings between sets having infinite domain but arbitrary image. The abstract concept corresponding to these concrete examples is that of a constellation, a concept first introduced by Gould and Hollings (who showed that the category of so-called inductive constellations is isomorphic to the category of left restriction semigroups).

Here we consider constellations in full generality, giving many examples. We characterise those small constellations that are isomorphic to constellations of partial functions, as well as those constellations that arise as (sub-)reducts of categories, and show that categories are nothing but two-sided constellations. We demonstrate that the naive notion of substructure can be captured within constellations but not within categories. We show that every constellation P gives rise to a category $\mathcal{C}(P)$, its "canonical extension", in a simplest possible way, that P is a quotient of $\mathcal{C}(P)$ obtained by factoring out a so-called canonical congruence, and that many familiar concrete categories may be constructed from simpler quotient constellations in this way. A correspondence between constellations and categories equipped with a canonical congruence is established.

This is joint work with Victoria Gould.

12. Heyting algebras with operators

Christopher Taylor (La Trobe University)
Mr Christopher Taylor

16:00 Tue 28 June 2016

It is well-known that congruences on a Heyting algebra are determined by filters on the underlying lattice. If an algebra $\mathbf{A}$ has a Heyting algebra reduct, it is of natural interest to characterise the filters that correspond to congruences on $\mathbf{A}$. Such a characterisation was given by Hasimoto, calling them *normal filters*. When normal filters can be described using a single unary term, many useful properties come to life. In general, a unary term that determines normal filters will be called a *normal filter term*. The traditional example comes from boolean algebras with operators (BAOs).

An algebra

$$\mathbf{B} = \langle B; \vee, \wedge, \neg, \{f_i \mid i \in I\}, 0, 1\rangle$$

is a *boolean algebra with (dual) operators* if

$$\langle B; \vee, \wedge, \neg, 0, 1\rangle$$

is a boolean algebra, and for each $i \in I$, the operation f_i is a unary normal operator, i.e., f_i is a map satisfying $f_i 1 = 1$ and $f_i(x \wedge y) = f_i x \wedge f_i y$. Conventionally, a BAO is defined dually, but it turns out that meet-preserving operations are more natural for Heyting algebras. If $\mathbf{B}$ is of finite type, then congruences on $\mathbf{B}$ are determined by filters closed under the map d, defined by

$$dx = \bigwedge\{f_i x \mid i \in I\}.$$

This is easily generalised to the case that each f_i is of any finite arity. Hasimoto gave a construction which generalises the term above to Heyting algebras equipped with an arbitrary set of operations. The construction does not apply in all cases, and even when it does, it does not necessarily produce a term function on the algebra. Having said that, Hasimoto proved that his construction guarantees a normal filter term for Heyting algebras with operators.

In this talk, we will extend Hasimoto's constraints to provide normal filter terms for a wider class of algebras. We will also speak about double-Heyting algebras, for which it is not known if Hasimoto's construction applies. Despite this, they are known to possess a normal filter term by a result of Sankappanavar. Finally, we will see how this can be used to prove that, for dually pseudocomplemented Heyting algebras, a variety $\mathcal{V}$ is semisimple if and only if $\mathcal{V}$ is a discriminator variety.

13. Explicit methods to compute number-theoretic objects

Shunichi Yokoyama (Kyushu University)
Dr Shunichi Yokoyama

15:00 Tue 28 June 2016

We survey explicit methods in number theory (especially in arithmetic geometry) and their implementations in computer algebra systems. In particular, we focus on the computational theory of elliptic curves and their implementations in Magma (born in Australia) that uses categorical structures to handle these curves. If time permits, we explain some methods to compute rational points (and integral points) on elliptic curves over several fields.

timetable

	Tuesday 28	Wednesday 29
9:00		
9:30	Johnson	Renz
10:00		
10:30	MORNING TEA	
11:00	Stokes	Mizoguchi
11:30		
12:00	Guttmann	Goldblatt
12:30		
13:00	LUNCH	
13:30		
14:00		
14:30	Elder	Bunder
15:00	Yokoyama	Jackson
15:30	AFTERNOON TEA	
16:00	Taylor	
16:30	Davey	
17:00	Kowalski	
17:30		
18:00	DINNER	